nth derivative of

$\log(ax+b)$

Let $y = \log(ax+b)$

$$y_1 = \frac{a}{(ax+b)} = a(ax+b)^{-1}$$

$$y_2 = (-1)\,a^2(ax+b)^{-2} = (-1)1!\,a^2(ax+b)^{-2}$$

$$y_3 = (-1)^2\,1\cdot 2\,(ax+b)^{-3}a^3 = (-1)^2 2!\,a^3(ax+b)^{-3}$$

$$y_n = (-1)^{n-1}(n-1)!\,a^n(ax+b)^{-n} = \frac{(-1)^{n-1}(n-1)!\,a^n}{(ax+b)^n}$$

i.e., $D^n[\log(ax+b)] = \dfrac{(-1)^{n-1}(n-1)!\,a^n}{(ax+b)^n}$

If $a=1$ and $b=0$ then $y = \log x$ and $y_n = \dfrac{(-1)^{n-1}(n-1)!}{x^n}$

$$y = \frac{x}{x^2-5x+6}$$

Let $\dfrac{x}{x^2-5x+6} = \dfrac{x}{(x-2)(x-3)} = \dfrac{A}{(x-2)} + \dfrac{B}{(x-3)}$

$$x = A(x-3) + B(x-2)$$

Put $x=3 \Rightarrow B=3$.

Put $x=2 \Rightarrow -A = 2 \Rightarrow A = -2$

$$\therefore y = \frac{-2}{x-2} + \frac{3}{x-3} = \frac{3}{x-3} - \frac{2}{x-2}$$

Differentiating n times we get

$$y_n = \frac{3(-1)^n n!}{(x-3)^{n+1}} - \frac{2(-1)^n n!}{(x-2)^{n+1}}$$